Gluons in small-x_B deep-inelastic scattering

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Abstract

The existing data obtained in deep-inelastic scattering (DIS) experiments in the kinematical region $x_B \lesssim 10^{-2}$ and $Q^2 \geq 5$ GeV^2 are examined to gain further insight into the dynamics of interacting gluons. It is shown that not only the regularities observed in diffractive scattering and those found in large-transverse-momentum jet production, but also the striking features of the structure function $F_2(x_B, Q^2)$ observed in normal DIS-events in this kinematical region can be understood as direct consequences of self-organized criticality. The results explicitly demonstrate the usefulness of concepts and methods of Complex Sciences in understanding the various striking features observed in high-energy collision processes in which "soft" gluons play the dominating role.

Deep-inelastic lepton-nucleon scattering utilizing electron, muon, and neutrino beams is an extremely useful tool in studying the structure of matter and its interactions. Since the operation of HERA, the first electron-proton collider, the kinematical region of such experiments has been significantly extended. In terms of the difference (q) between the initial and the final four-momenta (k and k') of the electron, and the four momentum (p) of the proton, the kinematical region in which data are now available is: $0 \leq Q^2 \leq 9 \cdot 10^4$GeV^2 and $10^{-5} \leq x_B \leq 1$, where $Q^2 \equiv -q^2$, and $x_B \equiv -q^2/(2pq)$.

Physics of deep-inelastic lepton-nucleon scattering (DIS) in the "small-x_B region

($x_B \lesssim 10^{-2}$, say)" for sufficiently large Q^2-values plays a very special role. In fact, "Small-x_B Physics" has become one of the most active fields in Particle Physics since the beginning of the 1990's. There are a number of reasons why this field has become so attractive. Some of them are listed below:

(α.1) It has been reported, soon after HERA started to deliver luminosity, by ZEUS-Collaboration[1] and H1-Collaboration[2] that, in this kinematical region, the structure function F_2 rises towards smaller x_B, with the strength of the rise increasing with Q^2[3]. Taken together with the fact that the valence quarks contribute very little to $x_B \lesssim 10^{-2}$, the above-mentioned experimental results[1−3] imply, not only that *there are many gluons in this region*, but also that *the interactions of such gluons play a significant role.* Now, since in accordance with QED and QCD, only charged particles namely the seaquark(s) and/or antiseaquark(s) can be "directly seen" by the virtual photon $\gamma^\star$, the interaction which are responsible for such pair-creation and pair-annihilation processes cannot be neglected at the moment when $\gamma^\star$ "sees something" from the gluon system. This means, in contrast to the kinematical region $x_B \gg 10^{-2}$ where preexisting valence quarks dominate, it is *not possible* to consider as first-order-approximation the struck quark or antiquark as a parton (which is *by definition* free) and to improve the approximation by subsequently taking the relevant Feynman diagrams into account. This means[3], theoretical assumptions (which specify e.g. which Feynman diagrams should be chosen, whether those leading to higher twists should be included, etc.) are needed as input — that is before the optimum set of parameters in the gluon PDF's (Parton Distribution Function) can be determined by fitting them to the experimental data[3].

(α.2) It is seen[3], that the above-mentioned extension of the parton idea with the help of pQCD and empirically determined PDF's indeed work well in reproducing the existing data[3]. Beautiful fits can be found[3] for all the measurable quantities inside as well as outside the small-x_B-region ($x_B \lesssim 10^{-2}$). While such fits must be considered as excellent *descriptions* of the existing experimental data[3], questions such as the following can, and should nevertheless be raised[3]: Is this *the only way* to describe Nature at the basic level of matter where experimental information can be obtained by performing high-energy collision processes at

relatively large momentum transfer (the so-called "hard" or "semi-hard" processes)? With such a large number of adjustable parameters in the theory, *how much can we learn about Nature* by comparing data with *fitted* curves? The reason why such questions deserve to be discussed can be readily seen by recalling what has happened in connection with the exciting discussions about the possible quark-substructure a few years ago: The excitement *began* when CDF Collaboration observed[4] a deviation between their data and a prediction based on pQCD and the then best version of PDF's. The excitement *ended* when a group of pQCD- and PDF-experts showed[5] that *perfect agreement* with the data can be achieved *by readjusting* some PDF-parameters concerning the gluons. Having seen this example, we can certainly understand why more and more people are now ready to adopt the view-point mentioned by Cooper-Sarkar, Devenish and de Roeck in their review article[3]: "Finally there is always the nagging doubt that the freedom to choose a fairly arbitrary function of x for the input parton distributions may be hiding the real breakdown of the standard description." It is perhaps worthwhile to compare the situation here with a well-known example in a different branch of science: While everybody agrees that earthquake is catastrophic, people may debate on "What is more useful in *understanding* Nature? Is it more useful to parameterize everthing we know about earthquakes, insert it as input, and make use of the largest and fastest computer to "predict" when, where, and how the next earthquake comes? Or, is it more useful to discover something like the Gutenberg-Richter law[6] although it cannot be directly used to make "predictions"?

(β) Events with large rapidity gaps (LRG) have been observed in this kinematical region[7]. This observation shows that *inelastic diffractive scattering* can take place not only in hadron-hadron collisions, but also in deep-inelastic scattering processes. This observation gives rise to a number of questions: Are the mechanisms which lead to such diffractive scattering processes in different reactions related to one another? In particular, if it is indeed "the exchange of colorless objects" which is responsible for diffractive scattering in hadron-hadron collisions, is it *the same kind of "colorless objects"* which is also responsible for diffractive scattering in deep-inelastic lepton-hadron scattering processes[7,3]? What *are* such "colorless

objects"? Can the existence of such objects be understood in terms of QCD? What is the relationship between such "colorless objects" and the interacting soft gluons mentioned in (α)?

(γ) In normal DIS-events, the observed x_B- and Q^2-dependence of F_2 mentioned in (α) and the relationship between these two properties can be well-described *quantitatively* by a simple empirical formula proposed by Haidt[8]

$$F_2(x_B, Q^2) = a + m \log \frac{Q^2}{Q_0^2} \log \frac{x_{B0}}{x_B}, \tag{1}$$

where the values of the constants are $a = 0.074$, $m = 0.364$, $x_{B0} = 0.074$, $Q_0^2 = 0.5$ GeV^2. As can be readily seen[8,9], this empirical rule of Haidt[8] indeed gives a very good description of the data[3] in the region $x_B \lesssim 10^{-2}$, for sufficiently large values of Q^2 ($Q^2 \geq 5$ GeV^2, say)! Is this empirical finding merely a parameterization where its simplicity is nothing else but a happy coincidence? Should this remarkably simple empirical fact about $F_2(x_B, Q^2)$ be ignored? Or, is it worthwhile to ask questions such as the following: Can this empirical formula be understood in terms of QCD? What is the relationship between this formula and the interacting soft gluons which dominate the small-x_B region? What is the relationship between this formula and the fact that LRG events exist in the small-x_B region?

The questions mentioned in (α), (β), and (γ) are closely related to one another — so close that they should have been discussed at the same time. But, for reasons which will be clear later on (see iv, v, and vi below), our research began with the facts and questions associated with those discussed in (β):

In a recent Letter[11] and a subsequent longer paper[12], we proposed that the "colorless objects" which manifest themselves in LRG events are color-singlet gluonic BTW-avalanches due to self-organized criticality (SOC)[13,14], and that optical-geometrical concepts and methods are useful in examining the space-time properties of such objects. The theoretical arguments and experimental facts which support the proposed picture can be summarized as follows:

(i) The characteristic properties of the gluons — especially the local gluon-gluon coupling

prescribed by the QCD Lagrangian, the confinement, and the non-conservation of gluon numbers — strongly suggest that systems of interacting soft gluons are *open, dynamical, complex* systems with *many degrees of freedom*, and thus in such systems colorless and colored gluon-clusters in form of BTW-avalanches (see below) can be formed.

(ii) It has been observed by Bak, Tang, and Wiesenfeld (BTW)[13] that a wide class of *open, dynamical, complex systems, far from equilibrium* evolve into self-organized critical states, and local perturbations of such critical states may propagate like avalanches caused by domino effects over all length scales. Such a long-range correlation effect eventually terminates after a time-interval T, having reached a final amount of dissipative energy, and having effected a total spatial extension S. The quantity S is called by BTW[13] the "size", and the quantity T the "lifetime" of the "avalanche" and/or the "cluster". It is observed[13,14] that there are many such open dynamical complex systems in the macroscopic world, and that the distributions D_S of S and the distribution D_T of T of such BTW avalanches/clusters obey power laws: $D_S(S) \propto S^{-\mu}$ and $D_T(T) \propto T^{-\nu}$, where μ and ν are positive real constants. Such characteristic behaviors are known[13,14] as the "fingerprints of SOC".

(iii) In order to see whether SOC and thus BTW-avalanches can also exist in microscopic systems at the level of quarks and gluons, a systematic analysis[12] of the data[3] for diffractive deep-inelastic scattering (DIS) has been performed. The results of the analysis can be summarized as follows: The "SOC-fingerprints" indeed exist in diffractive DIS in the small-x_B region ($x_B \lesssim 10^{-2}$) where interacting soft gluons play the dominating role. The size and the lifetime distributions of the BTW-avalanches observed in such processes indeed show power-law behaviors and the exponents are approximately -2. That is

$$D_S(S) \propto S^{-\mu}, \quad \mu \approx 2 \ , \tag{2}$$

$$D_T(T) \propto T^{-\nu}, \quad \nu \approx 2. \tag{3}$$

which is true in all Lorentz frames. The observed power-law behavior implies in particular that colorless gluon clusters are spatiotemporal complexities which have *neither* a typical size, *nor* a typical lifetime, *nor* a typical static structure.

(iv) The usefulness of the proposed SOC picture has been demonstrated in [11] and in [12]. It is shown in particular that simple analytical formulae for the differential cross-sections $d\sigma/dt$ and $d^2\sigma/dtd(M_x^2/s)$ can be derived for inelastic diffractive scattering, not only for small-x_B DIS and for photo-production but also for proton-proton and antiproton-proton collisions. It has been pointed out that *color-singlet* gluon clusters ($c_0^\star$) can be readily examined[11,12] experimentally in inelastic diffractive scattering processes, because the interactions between the struck $c_0^\star$ and any other *color-singlets* are of Van der Waal's type which are much weaker than color forces at distances of hadron-radius. Thus not very much momentum need to be transfered to a $c_0^\star$ by the projectile (which can be a $\gamma^\star$, a γ, a p or a $\bar{p}$) in order to "knock it out of the mother proton". For this reason, by considering inelastic diffractive scattering[11,12], we were able to check the existence and the properties of the *color-singlet* gluon-clusters.

(v) After having seen[11,12] the existence of SOC-fingerprints, and having demonstrated[11,12] the usefulness of such concepts and methods in Particle Physics by confronting them with the large body of experimental data[15] on inelastic diffractive scattering where the color-singlet and only the color-singlet and only the color-singlet gluon-clusters can be calculated, we discussed as the next step, the question: "What can we say about those gluon-clusters which are *not* color-singlets?" Here, we have to recall that, due to the (experimentally observed) SU(3) color-symmetry, most of such gluon-clusters are expected to carry color quantum numbers; that is, they are color-multiplets (instead of singlets) which will hereafter be denoted by $c^\star$'s. On the other hand, in accordance with the (experimentally confirmed) characteristic features of the BTW theory[13,14], *the existence of SOC fingerprints in gluon systems cannot, and should not, depend on the dynamical details of their interactions — in particular not on the intrinsic quantum numbers they exchange during the formation process.* This means in particular, that the size (S) distribution $D_S(S)$ and the lifetime (T) distribution $D_T(T)$ of the colored gluon clusters (the $c^\star$'s) should not only exhibit power-law behavior, but also have the same power as that found for color-singlet counterparts in diffractive scattering processes[11,12]! Is the existence of SOC in open dynamical complex

systems *indeed so general*, and its characteristic features *indeed so universal*? Can such expectations be *checked experimentally*?

(vi) The questions raised in item (v) has been discussed in a more recent Letter[16] where in particular the following has been pointed out: Similar to individual quarks (q's) or antiquarks ($\bar{q}$'s), colored gluon-clusters ($c^\star$'s) can also be "knocked out" of the mother proton p by a projectile, *provided that the corresponding transfer of momenta is large enough.* However, in contrast to the knocked-out q's or $\bar{q}$'s (in usual DIS-events), the knocked-out $c^\star$'s may or may not have "color lines" connected to the remnant of the proton — depending on the color-quantum number carried by the final state of the struck $c^\star$, after it absorbs the projectile which in proton-proton (pp) or proton-antiproton ($p\bar{p}$) collisions can be a quark or an antiquark or a gluon. Such knocked-out $c^\star$'s manifest themselves in form of hadronic jets. In fact, it has been explicitly shown in[16] that the observed high-E_T-jets in $\bar{p}p$ collisions can be described by the proposed SOC-picture. The same idea and the same method can be applied to lepton-induced DIS in which the large momentum transfer is delivered by the virtual photon $\gamma^\star$. The obtained results will be shown elsewhere.

We note in deriving the results presented in[16] the following properties of the colored gluon clusters in form of BTW-avalanches have been explicitly taken into account:

First, because of the universality and the robustness of SOC, the formation processes and the properties of the BTW-avalanches, in particular the SOC-fingerprints are expected to be *independent* of the intrinsic quantum number they carry. Having these and the following arguments in mind, we conclude that the size- and the lifetime-distributions of the colored gluonic BTW-avalanches are expected to be *the same* as those for colorless ones which have already been experimentally examined in diffractive scattering. It is useful to note that, inside a BTW-avalanche every constituent is in general interacting with more than one of its neighboring constituents, in fact, everyone of them is interacting directly or indirectly with everyone else through color forces. Hence, interaction between a BTW-avalanche and an incident space-like photon $\gamma^\star$ (or a hadron h) is weaker than the (average) interaction between the constituents of the BTW-avalanche. This means, in a collision process between

$\gamma^\star$ (or h) and a BTW-avalanche the latter acts as entire object — *independent* of the fact whether the interaction between the struck BTW-avalanche $c^\star$ and its neighbors is of Van der Waal's type. In other words, *the question whether a BTW-avalanche (either colorless or colored) can be knocked-out depends solely on the momentum-transfer it obtains in the collision process.*

Second, since gluonic BTW-avalanches are spatiotemporal complexities which have neither a typical size, nor a typical lifetime, nor a static structure, the scattering process between the virtual photon $\gamma^\star$ and the struck avalanche $c^\star$ is *not* a process in which "the electromagnetic structure of that avalanche" is being probed! The $\gamma^\star$ interacts within the allowed time interval with more than one of the charged constituents of $c^\star$, where the dynamical details about the individual subprocesses are rather unimportant. Roughly speaking, *the role played by $\gamma^\star$ is simply to deliver a sufficient amount of momentum-transfer to the gluonic avalanche $c^\star$ such that the entire $c^\star$ can be knocked-out of the mother proton.*

Let us now turn our attention to the majority of normal DIS-events, in which the virtual photon $\gamma^\star$ encounters a gluonic avalanche $c^\star$. We note, based on the facts mentioned above, it is important to know the probability for a $\gamma^\star$ to meet such a $c^\star$ of size S and lifetime T; and this can be immediately written down as the product of $D_S(S)D_T(T)$ and ST which is the space-time volume of the spatiotemporal complexity $c^\star$. Since $F_2(x_B \lesssim 10^{-2}, Q^2 \geq 5\ \mathrm{GeV}^2)$ is in fact nothing else but the total probability for the above-mentioned interaction in the given kinematical range to take place, we need to collect all those terms which contribute to the total probability. In doing so, we are led to the conclusion:

$$F_2(x_B, Q^2) = \\ N \int_{S_{min}}^{S_{max}} dS \int_{T_{min}}^{T_{max}} dT\, D_S(S) D_T(T) S T\,, \tag{4}$$

where $D_S(S)$ and $D_T(T)$ are given by Eqs.(2) and (3) respectively, N is a normalization constant, and the integration limits are functions of x_B, Q^2, and $P \equiv |\vec{P}|$:

$$S_{max} = x_{BO} P, \qquad S_{min} = x_B P, \tag{5}$$

$$T_{max} = \frac{4P}{Q_0^2}\frac{x_B}{1-x_B}, \quad T_{min} = \frac{4P}{Q^2}\frac{x_B}{1-x_B}. \tag{6}$$

The yet undetermined constants x_{B0} and Q_0^2 can be estimated theoretically (see below).

The facts and arguments which lead us to Eqs.(5) and (6) are the following: Since whatever the charged objects hit by $\gamma^\star(x_B, Q^2)$ may be, they are part of one of the BTW-avalanches which dominate the small-x_B region. S_{max} and S_{min} is proportional to the maximum and the minimum amount of dissipative energy this particular BTW-avalanche can carry. That is, they are given by Eq.(5), together with $x_{B0} \gtrsim 10^{-2}$ [17]. From Eqs.(4) and (5) we see, as expected, that the largest contribution for $F_2(x_B, Q^2)$ comes from the avalanches with the smallest size. Next, we consider the interaction-time τ_{int} of such a collision event in a proper (e.g. c.m.) Lorentz-frame: This can be estimated with the help of the Uncertainty Principle by calculating q^0 (of $q \equiv k - k'$), where the result is[11,12] $\tau_{int} \equiv 1/q^0 = 4|\vec{P}|Q^{-2}x_B(1-x_B)^{-1}$. The lower limit of T is determined by the requirement that the encountered BTW-avalanche has to live long enough to been "seen" by the virtual photon $\gamma^*(x_B, Q^2)$. The upper limit T_{max} is determined by the requirement that the charged object(s) which carries (carry) x_B has to be recognizable by $\gamma^\star$ (in the sense that $\gamma^\star$ should be able to find out whether they are part of the gluon-cluster of size S and lifetime T). Hence the resolution power (in the transverse directions, $1/Q^2$) of $\gamma^\star$ has to be sufficiently large (that is $Q^2 > Q_0^2$).

By inserting Eqs.(2), (3), (5), and (6) into Eq.(4) we obtain Eq.(1) where m is related to the normalization constant N in Eq.(4), and a can be interpreted as the averaged value of the contributions from the valence quarks in the small-x_B region[18]. In other words, the empirical formula proposed by Haidt[8], as shown in Eq.(1), turns out to be a natural consequence of the proposed[11,12,16] SOC-picture for interacting soft gluons which dominate small-x_B deep-inelastic scattering processes. However, while Haidt, from an experimental point of view, tried to extend the formula (1) to (if possible) all values of x_B, we see here, that, from a theoretical standpoint, it is expected to be valid only in the small-x_B region. The comparison with the experimental data is shown in Fig. 1.

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15. See experimental papers quoted in[11] and[12]. Our apology to the authors of these papers for not being able to repeat them here.

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17. We recall that in inelastic diffractive scattering [as mentioned in (β)], the size S (i.e. the dissipative energy) of the knocked-out color-singlet gluon-cluster in form of BTW-avalanche can be directly determined by measuring $x_P \approx M_x^2/s$ (see Refs.[11] and [12] for details). Taken together with the relation $x_B = \beta x_P$ where $0 \leq \beta \leq 1$ (see e.g. Refs.[7],[11] and [12]), we arrive at this conclusion.

18. An estimate has been made by using the momentum-distributions for valence quarks. It is interesting to see that the result is of the same order of magnitude as that given by Haidt [8].

FIGURES

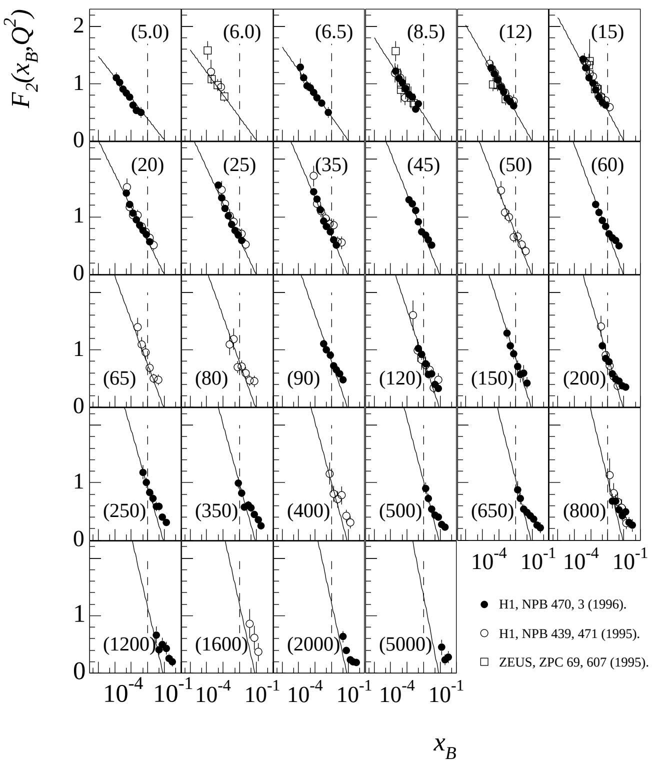

Fig. 1. The data for the proton structure function $F_2(x_B, Q^2)$ are taken from [10]. The solid lines are the calculated results by using the Haidt-formula [8] shown in Eq.(1). According to the theory presented in this paper, the formula Eq.(1) should only be valid for $x_B \lesssim 10^{-2}$. This is indicated by vertical dashed lines. The Q^2 values corresponding to each box are given in brackets.